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CALCULUS.

NOTE ON PROBLEM 84, BY DR. E. WOELFFING, STUTTGART, GERMANY.

The solution of question 84 can be found in some theorems proved by W. Merkelbach (*Ueber Rollkurven welche von einer Graden eingehüllt werden*, Diss. Marburg, 1881). The first of them is the following :

If a curve, C , rolls upon another curve, C' , and a point, P , in the plane of C describes a straight line L , and we make afterwards the curve C' roll upon the curve C , then a straight line, L , in the plane of C' will always pass through P (page 18 of the paper quoted).

Now (and this is the second theorem of Merkelbach) if a sinusoid rolls upon an ellipse, a straight line in the plane of the former passes through a focus of the latter (page 24) ; therefore, if the ellipse rolls upon the sinusoid, any one of the foci of the former will describe a straight line.

Stuttgart, Germany, July 19, 1899.

91. Proposed by GUY B. COLLIER, Schenectady, N. Y.

Find the area of a loop of the curve $r^2 \cos \theta = a^2 \sin 3\theta$. [From Hall's *Differential and Integral Calculus*].

I. Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; M. C. STEVENS, A. M., Professor of Mathematics, Purdue University, Lafayette, Ind.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; ELMER SCHUYLER, Reading, Penna.; and J. SCHEFFER, A. M., Hagerstown, Md.

The curve has two equal loops, one in the first and the other in the third quadrant.

The limits of θ are 0 and $\frac{1}{3}\pi$.

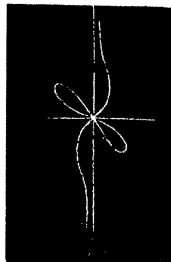
$$\begin{aligned}\therefore A &= \frac{1}{2} \int_0^{\frac{1}{3}\pi} r^2 d\theta = \frac{1}{2} a^2 \int_0^{\frac{1}{3}\pi} \frac{\sin 3\theta d\theta}{\cos \theta} \\ &= \frac{1}{2} a^2 \int_0^{\frac{1}{3}\pi} (4 \sin \theta \cos \theta - \tan \theta) d\theta = \frac{1}{2} a^2 (3 - 2 \log 2).\end{aligned}$$

II. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; and GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.

The shape of the curve is seen in the diagram.

The limits are evidently from 0° to 30° for a loop.

$$\begin{aligned}\text{Then } A &= \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int_0^{\frac{1}{3}\pi} \frac{a^2 \sin 3\theta}{\cos \theta} d\theta \\ &= \frac{a^2}{2} \int_0^{\frac{1}{3}\pi} \frac{3 \sin \theta - 4 \sin^3 \theta}{\cos \theta} d\theta = \frac{a^2}{2} \int_0^{\frac{1}{3}\pi} 3 \tan \theta d\theta \\ &\quad - 2a^2 \int_0^{\frac{1}{3}\pi} (\tan \theta - \sin \theta \cos \theta) d\theta\end{aligned}$$



$$= \frac{3a^2}{2} \log \sec \theta - 2a^2 \log \sec \theta + a^2 \sin^2 \theta \Big]_0^{\frac{1}{2}\pi} = a^2 \sin^2 \theta - \frac{a^2}{2} \log \sec \theta \Big]_0^{\frac{1}{2}\pi}$$

$$= \frac{3a^2}{4} - \frac{a^2}{2} \log 2.$$

[NOTE. In the figure, the loop in the fourth quadrant should be in the first, and the one in the second in the third.

92. Proposed by B. F. SINE, Principal of Capon Bridge Normal School, Capon Bridge, W. Va.

How much wood is taken from a log 12 inches in diameter, by boring a two-inch hole through the center, the axis of the hole being perpendicular to axis of log ?

I. Solution by G. B. M. ZERE, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J. SCHEFFER, A. M., Hagerstown, Md., and WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

$$V = 8 \iiint dx dy dz = \text{volume}.$$

The equation of the surface of the cylinder corresponding to the log is

$$x^2 + y^2 = 36 = R^2,$$

and the equation of the surface of the cylinder corresponding to the auger-hole is

$$x^2 + z^2 = 1 = r^2.$$

$$V = 8 \iiint y dx dz = 8 \int_0^r \int_0^{\sqrt{r^2 - x^2}} \sqrt{R^2 - x^2} dx dy.$$

$$\therefore V = 8 \int_0^r \sqrt{(r^2 - x^2)(R^2 - x^2)} dx$$

$$= \frac{8}{3} R^3 \{ [1 + (r/R)^2] E(r/R) - [1 - (r/R)^2] F(r/R) \}$$

$$= 576 \{ (1 + \frac{1}{3}) E(\frac{1}{3}) - (1 - \frac{1}{3}) F(\frac{1}{3}) \}$$

$$= 576 \{ \frac{2}{3} E(\frac{1}{3}) - \frac{2}{3} F(\frac{1}{3}) \} \text{ cubic inches.}$$

$$\text{Also } V = 8 \int_0^r \sqrt{(r^2 - x^2)} \left[R - \frac{x^2}{2R} - \frac{x^4}{8R^3} - \dots - \frac{1.3.5.7 \dots (2n-1)x^{2n}}{2.4.6 \dots 2nR^{2n-1}} \right] dx.$$

$$\text{Now } \int_0^r \sqrt{(r^2 - x^2)} x^{2n} dx = \frac{1.3.5.7 \dots (2n-1)}{2.4.6.8 \dots (2n+2)} \pi r^{n+2}.$$

$$\therefore V = 4r^2 R \pi \left[\frac{1}{2} - \frac{r^2}{16R^2} - \frac{r^4}{128R^4} - \dots - \frac{(1.3.5.7 \dots 2n-3)^2 (2n-1)r^{2n}}{(2.4.6 \dots 2n)^2 (2n+2)R^{2n}} - \dots \right]$$

$$\therefore V = 24\pi \left(\frac{1}{2} - \frac{1}{672} - \frac{1}{16384} - \frac{1}{98304} - \dots \right) = 37.56784 \text{ cubic inches.}$$